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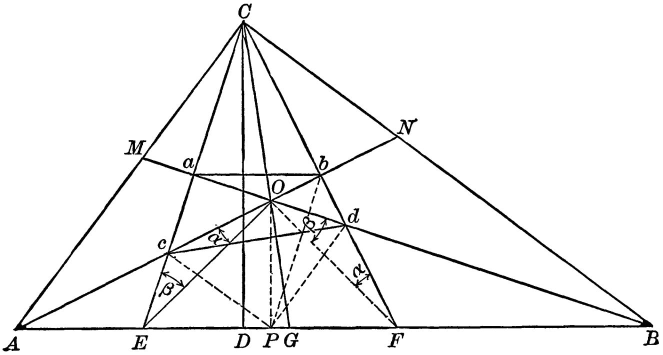
the trigonometric solution are $\cos 3\theta = -m$, $x = 2a\cos\theta$; if $|m|$ is nearly unity, the value of θ is not very accurately determined. The first of the examples cited above is near the border-line; five-place tables will give nearly as accurate results by the trigonometric method as by the approximation, but with more labor. [The roots by this method are -0.0721, 2.4326, 2.1394.]

II. Relating to Some Relations in a Right-angled Triangle.

By Albert Babbitt, University of Nebraska, Lincoln.

In the right-angled triangle ABC (with right angle at C) draw CD perpendicular to AB; also draw CE, CF, BM and AN bisectors of the angles ACD, BCD, ABC and CAB respectively.



We have then the following theorems:

Theorem I. *The bisector of the angle B is perpendicular to CE, and the bisector of the angle A is perpendicular to CF. Moreover, CE is bisected by the bisector of the angle B (in point a) and CF is bisected by the bisector of the angle A (in pt. b), and consequently ab is parallel to AB and is equal to $\frac{1}{2}EF$.*

Proof. Since

$$\underline{|bOd} = \frac{A}{2} + \frac{B}{2} = 45^\circ,$$

$$\underline{|bdO} = \frac{BCD}{2} + \frac{B}{2} = 45^\circ,$$

hence $\underline{|dbO} = 90^\circ$, and AN is perpendicular to CF. Similarly, it may be proved that BM is perpendicular to CE.

From the equality of the right-angled triangles FbA and AbC, we have $Fb = bC$, i. e., CF is bisected by the bisector of the angle A. Similarly, $Ea = aC$, and it follows at once that ab is parallel to AB and is equal to $\frac{1}{2}EF$.

Theorem II. *The bisectors CF and CE cut off on the hypotenuse AB segments AF and BE which are equal to the sides AC and BC respectively.*

Proof. From the equality of the right triangles FbA and AbC, it follows that

$AF = AC$, and from the equality of the right triangles BaE and BaC it follows that $BE = BC$.

THEOREM III. *The radius r of the circle inscribed in the triangle ABC is equal to $\frac{1}{2}EF = ab$.*

Proof. Since $CO = OF$ (ΔCOF is isosceles) and $CO = OE$ (ΔCOE is isosceles), $OE = OF$; and consequently EFO is an isosceles triangle. Moreover, $\underline{|FOE} = 2\,\underline{|FCE} = 90^\circ$. Whence, $OP \equiv r = \frac{1}{2}EF = ab$.

THEOREM IV. *The radius, $r = cd/\sqrt{2}$, i. e., the radius is equal to the side of a square whose diagonal is cd.*

Proof. From the similarity of the triangles aOb and dOc, we have

$$\frac{ab}{cd} = \frac{Oa}{Oc} = \frac{Ob}{Od} = \sin 45^\circ = \frac{1}{\sqrt{2}}.$$

Hence (Theorem III), $r = ab = \dfrac{cd}{\sqrt{2}}$.

THEOREM V. *The line of centers of the circles inscribed in the triangles ADC and BDC is perpendicular to the bisector of the right angle C, i. e., $cd \perp CG$.*

Proof. Since in the triangle cdC two altitudes cb and da pass through the point O, $CG \perp cd$.

THEOREM VI. *The centers of the circles inscribed in the triangles ADC and DBC, i. e., the points c and d are equidistant from the point P and their distances are equal to $OP = r$.*

Proof. From the equality of triangles OFd and OcE it follows that $OD = Ec$, and hence follows the equality of OPd and PEc and consequently $cP = Pd$. Moreover, $\underline{|OdF} = 180^\circ - (\alpha + \beta) = 180^\circ - 45^\circ = 135^\circ$. Hence, if a circle be drawn with P as the center and $OP = PF = r$ as the radius $\underline{|OdF}$ will be an inscribed angle and $Pd = cP = r$.

UNDERGRADUATE MATHEMATICS CLUBS.

EDITED BY R. C. ARCHIBALD, Brown University, Providence, R. I.

ADDRESS.[1]

By HERBERT E. HAWKES, Columbia University.

In the remarks which I have to make this morning, I do not propose to go into details regarding the history of Mathematics Clubs or to discuss the function of such clubs under ideal conditions. I wish rather to tell briefly of my own experience with such clubs which may possibly serve as a practical help to those who have conducted or are about to organize such organizations in their own institutions.

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[1] Delivered at the second summer meeting of the Mathematical Association of America, at Cleveland, Ohio, September 6, 1917.